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DETERMINATION OF THERMOREOLOGICAL CHARACTERISTICS AT MICROSCOPIC SCALE FROM EXPERIENCES ON THIN WALL TUBES

Vasile Marina, Viorica Marina[1*]
[1]Technical University of Moldova, 168, Stefan cel Mare str., Chisinau, Moldova
*Corresponding author: Viorica Marina, marina_viorica@yahoo.com

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Abstract. The relationships between the module of stress and strain tensor deviator, in the case of thermorheological processes, are modeled by using the structural model. It is shown that sub-element properties can be determined from some experiences on thin walls tubes loaded with axial forces and interior pressure. Because the constants and material functions for this material are unknown the loaded conditions with constant state parameters are required. If the stretching process is produced with a constant speed of axial strain for each material there is one constant report among axial and circumstantial stresses which ensures in isothermal processes state constant parameters.

Keywords: *Stress, strain, structure, rheology, thermo-mechanic, crystals, polycrystal*

Introduction

It is well-known that the disordered environment characteristic of most of the materials used in the technique is considered statistically homogeneous at macroscopic level. The minimal volume, which satisfies this requirement we will note with ΔV_0, but the surface which delimits it through ΔS_0. The volume element ΔV_0 is considered compound from an infinite number of structural sub-elements, which in their turn, contain the sufficient number of atoms, that the conception of the continuous environment also remains valid at sub-element level. The subelements are cinematically linked to each other and have simple, but different, thermorheological properties. Due to subelements that are endowed with simple thermorheological properties, they are determined on the basis of restricted number of experiences. Complex properties which building materials possess at macroscopical level are due to extremely complex interactions among the subelements. Thus the advantage of the structural model in comparison with theories and phenomenological models, proposed by different authors, consists in possibility of description under unit form of broad spectrum of thermomechanical phenomena, on the basis of small amount of experiences.

1. The methodology of transition from micro-stresses and strains to macro-stresses and strains

The thermomechanical magnitudes, which describe the behavior of materials depending on their structure and historical exterior action, are defined at two levels: macroscopic (conglomerate/system of subelements) and microscopic (subelement). Thus, we will use the following parameters at macroscopical level: t_{ij} - stress tensor; d_{ij} - strain tensor; $\sigma_{ij} = t_{ij} - \frac{1}{3} t_{nn} \delta_{ij}$ - stress tensor deviator; $\varepsilon_{ij} = d_{ij} - \frac{1}{3} d_{nn} \delta_{ij}$ - strain tensor deviator; σ - module of stress tensor deviator defined by the relationship $\sigma = \sqrt{\sigma_{ij}\sigma_{ij}}$, ε - module of strain tensor deviator, $\varepsilon = \sqrt{\varepsilon_{ij}\varepsilon_{ij}}$. At macroscopic level all magnitudes are noted with bars over them: $\bar{t}_{ij}, \bar{d}_{ij}, \bar{\sigma}_{ij}$.

If on the conglomerate's surface of subelements are homogeneous, then on the basis of equilibrium and Cauchy's geometrical equations in [1] R. Hill established the following fundamental relationships:

$$t_{ij} = \langle \bar{t}_{ij} \rangle = \frac{1}{\Delta V_0} \int_{\Delta V0} \bar{t}_{ij} dV \ , \tag{1}$$

$$d_{ij} = \langle \bar{d}_{ij} \rangle = \frac{1}{\Delta V_0} \int_{\Delta V0} \bar{d}_{ij} dV \ , \tag{2}$$

$$\langle \bar{t}_{ij} \bar{d}_{ij} \rangle = \langle \bar{t}_{nm} \rangle \langle \bar{d}_{nm} \rangle . \tag{3}$$

From „Eq.(1)" it results that macroscopic stresses are equal with average of microscopic stress and therefore by using „Eq.(2)" we obtain that macroscopic strains are equal with average of microscopic strains.

„Eq.(1) - (3)" are necessary, but not sufficient for construction of governing equations at conglomerate level on the basis of physic equations at subelement level. To achieve a complete system of equations new relationships are needed. From R.Hill relationships we can conclude, that volume means of stresses, strains and their scalar product (see „Eq.(1)") depends univocally on data surface ΔS . But not all microscopic variables have this specific property. In [2, 3] it has been proven, that for spherical tensors and deviators (see „Eq.(1)") the following relations may fail to hold:

$$\langle \sigma_{ij} \varepsilon_{ij} \rangle - \langle \sigma_{ij} \rangle \langle \varepsilon_{ij} \rangle \neq 0 , \tag{4}$$

$$\langle \sigma_0 \varepsilon_0 \rangle - \langle \sigma_0 \rangle \langle \varepsilon_0 \rangle \neq 0 . \tag{5}$$

In [3] the relations of type „Eq.(1), (6)" were named discordance. The discordance among macroscopic suitable values is carrier of information's about one string of structural subelements of composite material. In [2] it was postulated principle: *in all real interactions the discordances of microscopic values with their suitable macroscopic analogs the extreme values are obtained*:

$$\langle \bar{\sigma}_{ij} \bar{\varepsilon}_{ij} \rangle - \langle \bar{\sigma}_{ij} \rangle \langle \bar{\varepsilon}_{ij} \rangle \neq extr . , \tag{6}$$

$$\langle \bar{\sigma}_0 \bar{\varepsilon}_0 \rangle - \langle \bar{\sigma}_0 \rangle \langle \bar{\varepsilon}_0 \rangle \neq extr. \tag{7}$$

The second principle was obtained starting from the mechanisms of crystals deformation in polycrystalline conglomerate. The experimental researches demonstrate that there exists auto coordination of deformation processes among material particles from conglomerate.

In [3, 4] it was formulated the principle of medium ties, according to which, the interactions among sub-elements are formed only under medium ties influence. Starting with this principle and presentation of mentioned three R. Hill relationships under one single expression:

$$\left\langle \Delta\bar{t}_{ij} \Delta\bar{d}_{ij} \right\rangle = 0, \tag{8}$$

where $\Delta\bar{t}_{ij} = \bar{t}_{ij} - t_{ij}$, $\Delta\bar{d}_{ij} = \bar{d}_{ij} - d_{ij}$,

was postulated the principle

$$\Delta\bar{t}_{ij} \Delta\bar{d}_{ij} = 0. \tag{9}$$

According to „Eq.(9)": *the scalar product (interior) among fluctuations of stress and strain tensors are canceled in each subelement of conglomerate.*

If in „Eq.(9)" we decompose the stress and strain tensors in deviators and spherical tensors, we establish an additional fundamental relationship [4]:

$$\left(\bar{\sigma}_{ij} - \sigma_{ij}\right)\left(\bar{\varepsilon}_{ij} - \varepsilon_{ij}\right) = 3\left(\bar{\sigma}_0 - \sigma_0\right)\left(\varepsilon_0 - \bar{\varepsilon}_0\right). \tag{10}$$

From „Eq.(10)" it results, that in polycrystalline conglomerate, at sub-element level, any variation of stress/strain tensors deviators provokes spherical tensors variations. Due to these properties, we succeed to describe one string of thermomechanical properties, known from the experience, but can't be explained in another theories or models.

In order to obtain one complete system of equations in [3] it was postulated an additional principle: *for any thermomechanical processes the fluctuations of stress deviators are univocal functions of fluctuations of strain deviators*. In linear approximation we have:

$$\bar{\sigma}_{ij} - \sigma_{ij} = B_{ijnm}\left(\varepsilon_{ij} - \bar{\varepsilon}_{ij}\right), \tag{11}$$

in which fourth order tensor B_{ijnm} depends on structural factors at microscopical level and reflects the cinematic interaction among sub-elements in conglomerate.

On the basis of complete system of equations „Eq.(1) – (3), (6) or (7)", „Eq.(10) and (11)" there can be established the governing equations at macroscopical level, in the case when physical relationships at microscopical level are known.

2. The thermorheological properties of subelements

Hereinafter we will admit that the subelements are isotropic. In this case the equation of composition „Eq.(11)" is simplified and gets the form:

$$\bar{\sigma}_{ij} - \sigma_{ij} = 2Gb\left(\varepsilon_{ij} - \bar{\varepsilon}_{ij}\right), \tag{12}$$

where through G is noted shearing module at macroscopic level, but through b - intern parameter, which reflects the inhomogeneities of stress and strain states in the inner of conglomerate. If strain deviators in subelements and at macroscopic level are discomposed in deviators of strain reversible tensors - e_{ij} and irreversible - p_{ij}, namely:

$$\bar{\varepsilon}_{ij} = \bar{e}_{ij} + \bar{p}_{ij}, \qquad \varepsilon_{ij} = e_{ij} + p_{ij}, \tag{13}$$

than „Eq.(12)" can be written under more convenient shape

$$\bar{e}_{ij} - e_{ij} = m\left(p_{ij} - \bar{p}_{ij}\right), \qquad m = \frac{b}{b+1}. \tag{14}$$

In „Eq.(14)" only dimensionless values figurate and thus describing nonlinear processes, π - *theorem* is verified in automatic mode. On the basis of these values there can be obtained the direct relationship among reversible and irreversible strains.

Physical relationships for system with infinite number of sub-elements can be described by a single expression (proportional processes):

$$\bar{e}_{ij} = \frac{\left(\tau(\psi,\upsilon,\gamma) + a\bar{p}\right)p_{ij}}{p}, \qquad p = \sqrt{p_{ij}\,p_{ij}}, \tag{15}$$

where through τ is noted the elasticity limit of considered sub-element, ψ – the parameter of identification of subelements $(0 \le \psi \le 1)$, υ - the thermal strain, *a* – the work-hardening coefficient, γ - the parameter, which is equal to average value of speed of irreversible strain in subsystem of strained subelements after the elasticity limit:

$$\gamma = \frac{\sqrt{\dot{p}_{ij}\,\dot{p}_{ij}}}{\psi^*} = \frac{\dot{p}}{\psi^*}. \tag{16}$$

In „Eq.(16)", through ψ^* it was noted the actual weight of subelements loaded after the elasticity limit (subelements for which $0 \le \psi \le \psi^*$ are loaded after the elasticity limit, but subelements with values $\psi^* \le \psi \le 1$, continue to be required in the reversible field).

In the case of some proportional solicitations, „Eq.(14)" and „Eq.(15)" can be written as follow:

$$\bar{e} - e = m\left(p - \bar{p}\right), \tag{17}$$

$$\bar{e}(\psi) = \begin{cases} \tau(\psi,\upsilon,\gamma) + a\bar{p}, & \psi < \psi^* \\ \tau(\psi^*,\upsilon,\gamma), & \psi \ge \psi^* \end{cases}. \tag{18}$$

From „Eq.(17)" and „Eq.(18)" we obtain the relationships for sub-element characteristics:

$$\tau(\psi,\upsilon,\gamma) = e + mp(\psi,\upsilon,\gamma), \tag{19}$$

$$\psi = \frac{(a+m)p_{,e}}{1+mp_{,e}}, \ \ p_{,e} = \frac{\partial p}{\partial e}. \tag{20}$$

Taking into account „Eq.(20)” in „Eq.(16)” we find the relation for state parameter γ:

$$\gamma = \frac{\dot{e}+m\dot{p}}{a+m} = \frac{1-\chi}{b+\chi}\left(\frac{\dot{\sigma}}{2G}+b\dot{\varepsilon}\right), \quad \chi = \frac{a}{1+a}. \tag{21}$$

3. Solicitation conditions with constant state parameters: $\gamma = const., \upsilon = const.$

Since we operate with tensorial values in experience we will pass in „Eq.(21)”, from speed of stress and strain tensors deviators modules to respective tensorial components. In the examined case we have

$$\frac{\sigma_{ij}}{\sigma} = \frac{\varepsilon_{ij}}{\varepsilon} = r_{ij} = const. \tag{22}$$

If in „Eq.(21), (22)” we use the tensorial values, than we obtain:

$$\gamma = \frac{1-\chi}{b+\chi}\left(\frac{\dot{t}_{ij}-\dot{\sigma}_0\delta_{ij}}{2G} - b\frac{\dot{\sigma}_0}{K}\delta_{ij} + b\dot{d}_{ij}\right)\frac{1}{r_{ij}}, \langle i,j=1,2,3\rangle, \tag{23}$$

where *K* is compression module.

Let's examine experiences on tubes with thin walls, solicited for stretching and internal pressure. Axial stresses and strains we denote respectively by - t_{11}, d_{11}, and circumferential stresses and strains denote by - t_{33}, d_{33}. In this case γ parameters are determined from one of three formulas:

$$\gamma = \frac{1-\chi}{b+\chi}\left\{\left[\frac{\dot{t}_{11}}{2G}\left(1-\frac{1+c_3}{3}\left(1+b\frac{2G}{K}\right)\right)\right] + b\dot{d}_{11}\right\}\frac{1}{r_{11}}, \tag{24}$$

$$\gamma = \frac{1-\chi}{b+\chi}\left\{\left[\frac{\dot{t}_{11}}{2G}\left(1-\frac{1}{3}\left(1+\frac{1}{c_3}\right)\left(1+b\frac{2G}{K}\right)\right)\right] + b\dot{d}_{33}\right\}\frac{1}{r_{33}}, \tag{25}$$

$$\gamma = \frac{1-\chi}{b+\chi}\left\{\left[-\frac{\dot{\sigma}_0}{2G}\left(1+b\frac{2G}{K}\right)\right] + b\dot{d}_{22}\right\}\frac{1}{r_{22}}, \tag{26}$$

where

$$c_3 = \frac{t_{33}}{t_{11}}, \tag{27}$$

$$\sigma_0 = \frac{t_{11} + t_{33}}{3} = \frac{1 + c_3}{3} t_{11} \tag{28}$$

Form the required conditions we obtain the following values for r_{ij}:

$$r_{11} = \frac{t_{11} - \sigma_0}{\sqrt{(t_{11} - \sigma_0)^2 + (t_{33} - \sigma_0)^2 + \sigma_0^2}} = \frac{2 - c_3}{\sqrt{6(1 + c_2^2 - c_3)}}, \tag{29}$$

$$r_{11} = \frac{2c_3 - 1}{\sqrt{6(1 + c_3^2 - c_3)}}, \tag{30}$$

$$r_{22} = \frac{1 + c_3}{\sqrt{6(1 + c_3^2 - c_3)}}. \tag{31}$$

Analyzing relationships „Eq.(24) - (26)" we are convinced, that the signs and values of the expressions included in square brackets [ ] in the first two formulas, depend on the value of the parameter c_3. This is the situation we can use during the solicitation with $\gamma = const.$

Under laboratory conditions, experiments are easier to accomplish when $\dot{d}_{11} = const.$

The condition can be accomplished if in „Eq.(24)" we will admit

$$1 - \frac{1 + c_3}{3}\left(1 + b\frac{2G}{K}\right) = 0.$$

$$\gamma = b\frac{1 - \chi}{b + \chi}\frac{\sqrt{6(1 + c_3^2 - c_3)}}{2 - c_3} d_{11} \tag{33}$$

Thus, under the conditions of the relationship „Eq.(32)", external stress indicator for $\gamma = const.$ is strain with $\dot{d}_{11} = const.$ Solving „Eq.(32)" to c_3, we find the position of the trajectory in the space t_{11}, t_{33}, which will correspond to intern parameter b given:

$$c_3 = \frac{2 - b\frac{2G}{K}}{1 + b\frac{2G}{K}} \tag{34}$$

Knowing the variation limits of the parameter $0 \le b \le \infty$, from „Eq.(34)" we set the size limits of the magnitudes of c_3.

If *b=0* (the homogeneous stress state), than from „Eq.(34)" result that $c_3 = 2$, but for $b = \infty$ (the homogeneous strain state), $c_3 = -1$. Therefore magnitude c_3, based on which the report among stress circumferential and axial values of tensors is established, varies in the following range:

$$-1 \le c_3 \le 2. \tag{35}$$

In this interval there can be realized solicitations under thin walls pipes if $\gamma = const.$ for any possible scheme of cinematic interactions among subelements.

Expressing in „Eq.(34)" *b* through *m*, but the report $\frac{2G}{K}$ - through Poisson coefficient, we will find:

$$c_3 = \frac{2(1+\upsilon)-3m}{1+\upsilon-3m\upsilon}. \tag{36}$$

The laws of variations of report among stresses $\frac{t_{33}}{t_{11}} = c_3$, for different cinematic interactions schemes in subelements system, in function of Poisson coefficient values $0 \le \upsilon \le 0{,}5$, are presented in "Figure 1".

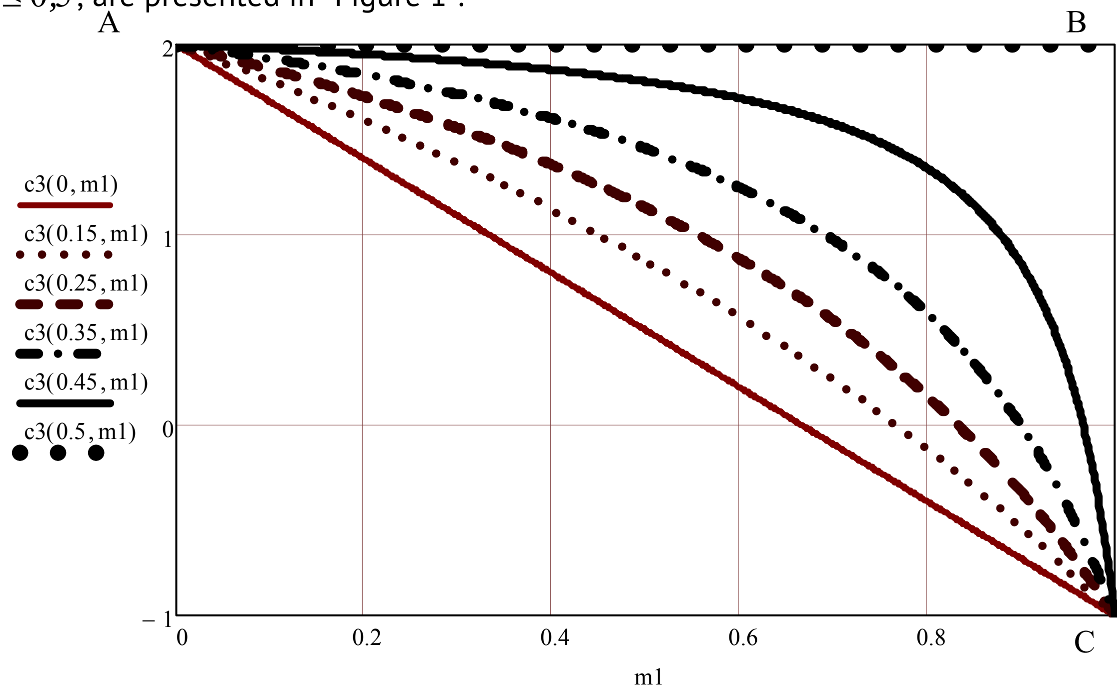

Figure 1. The laws of variations of report among stresses for different cinematic interactions schemes in function of Poisson coefficient values

From "Figure 1" we observe that c3 curves for all possible values of υ coefficient are located inside of ABC triangle.

Results and discussion

Once the loading conditions with constant state parameters are established, we mention the material function:

$$\varepsilon = f\left(\frac{\sigma}{2G}, \gamma, \upsilon\right) = f(e, \gamma, \upsilon), \tag{37}$$

which can be presented as follows:

$$p(e,\gamma,\upsilon)= f(e,\gamma,\upsilon)-e. \tag{38}$$

By introducing „Eq.(38)” in „Eq.(19)” and „Eq.(20)”, we find the relationships of subelements characteristics in the following parametric form:

$$\psi = \frac{(a+m)\left(f(e,\gamma,\upsilon)_{,e}-1\right)}{1+m\left(f(e,\gamma,\upsilon)_{,e}-1\right)}, \tag{39}$$

$$\tau(\psi,\gamma,\upsilon)=e(1-m)+mf(e,\gamma,\upsilon). \tag{40}$$

Conclusions

The rheological state parameter γ in subelements of tube with thin walls, subject to action of some stretching force and interior pressure, doesn’t change during the experience, if the condition $\dot{d}_{11}=const.$ is satisfied in the action period and ratio $\frac{t_{33}}{t_{11}}=c_3$ is in concordance with „Eq.(36)”. From the set of interaction schemes in the subelements, which concomitant reflects the no homogeneities of stress and strain state we obtain a special case for $m=\frac{2}{3}(1+\upsilon)$.

The experience with $\gamma=const.$ for this value of *m*, according to the „Eq.(36)”, corresponds to an axial load $(c_3=0, t_{33}=0)$, with strain axial constant speed.

We also mention that from the „Eq.(21)” results, that for any type of loading, in the moment of reaching the threshold of material passing from reversible to irreversible domain the state parameter γ is proportional with speed of strain tensor deviator module. So, through this effect in any loading conditions at macroscopic level the continuum passing from reversible to irreversible material state is assured.

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